

Handout (Week 4)

Natural Language Arguments and Formal Arguments

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Goal of this handout: use the formal language L to evaluate arguments in natural language.

Reading guide for completing the homework

- To solve homework problems 1-10, you will probably have to read all of sections 1.1 and 1.2.

Summary of handout

- Section 1.1: I present the rigorous definitions of valid arguments, invalid arguments, sound arguments, and unsound arguments in English.
- Section 1.2: I present an example of a valid argument, and I present an example of a sound argument.
- Section 1.3: I present some more examples.
- Section 2: I explain why the notions of validity and soundness are so important.

1 Validity in Natural Language

In this section, I explain how arguments in L can be used to determine whether or not certain natural language arguments are valid. Then I present some examples of this.

1.1 Final Definition of Validity for English Arguments

Recall that in the handout from week 1, I presented preliminary definitions of validity, invalidity, soundness, and unsoundness, for arguments in English. The definition of validity for arguments in L can be used to refine, and precisify, those preliminary definitions. In this section, I explain how.

The new definitions will rely on translations. In particular, in those definitions, I will refer to the ‘translation’ of an argument in English. The translation of such an argument is the best representation, in L , of the sentences in that argument. For example, suppose that the original argument is as follows.

1. It is raining.
2. If it is raining, then there are clouds in the sky.
3. There are clouds in the sky.

Then the translated argument is this.

1. p
2. $p \rightarrow q$
3. q

where ‘ p ’ represents the sentence “It is raining”, and ‘ q ’ represents the sentence “There are clouds in the sky”.

Now for the final, fully rigorous definition of valid arguments in English.

Definition: Valid Argument (in English, final)

A ‘valid argument’ in English is an argument in English with the following property: when that argument is translated into L , the translated argument is valid.

Here is the rough idea behind this definition. Take an argument in natural language. Translate it into an argument in L : in other words, translate each sentence of the natural language argument into a sentence of propositional logic. Then see if that translated argument is valid in L . If so, then the above definition says that the original, natural language argument is valid too. If not, then not.

Here are the definitions of invalid arguments, sound arguments, and unsound arguments in English.

Definition: Invalid Argument (in English, final)

An ‘invalid argument’ in English is an argument in English which is not valid.

Definition: Sound Argument (in English, final)

A ‘sound argument’ in English is a valid argument in English whose premises are all true.

Definition: Unsound Argument (in English, final)

An ‘unsound argument’ in English is an argument in English which is not sound.

1.2 Examples

In this section, I show how to use the definitions from Section 1.1 to determine whether certain arguments in English are valid or sound.

To start, consider once more the English argument below.

1. It is raining.
2. If it is raining, then there are clouds in the sky.
3. There are clouds in the sky.

Call this the ‘rain argument’.

Here is the argument which results when the rain argument is translated into L .

1. p
2. $p \rightarrow q$
3. q

Recall that in this translation, ‘ p ’ represents the sentence “It is raining”, and ‘ q ’ represents the sentence “There are clouds in the sky”.

As I showed in the handout from week 3, this argument is valid. Here is its truth table.

p	q	p	$p \rightarrow q$	q
T	T	T	T	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

As a simple check confirms, for each assignment of truth values to sentence letters which makes *all* of the premises in the argument true, that assignment *also* makes the conclusion true. For there is just one assignment of truth values to sentence letters which makes all of the premises in the argument true: the first assignment; that is, the one in the second row. Each other assignment makes at least one premise in the argument false. And that first assignment also makes the conclusion true. So by the definition of validity for arguments in L , the above argument is valid.

Therefore, by the definition of validity for arguments in English, the rain argument is also valid. In other words, the rain argument is valid because when translated into L , the resulting formal argument is valid. So the original, untranslated argument is valid too.

Let us now explore an example of a sound argument. Consider the argument below.

1. Iron oxide is either red or yellow.
2. Iron oxide is not yellow.
3. Iron oxide is red.

Call this the ‘iron argument’.

To determine whether or not the iron argument is sound, we must do two things. First, we must determine whether or not it is valid. Second, we must determine whether or not its premises are true. If it is valid, and it has true premises, then it is sound. Otherwise, it is unsound.

So to start, let us see whether the iron argument is valid. To do that, we must translate the iron argument into an argument in L . The result is below.

1. $p \vee q$.
2. $\neg q$
3. p

In this translation, ‘ p ’ represents the sentence “Iron oxide is red”, and ‘ q ’ represents the sentence “Iron oxide is yellow”.

To determine whether or not this argument is valid, construct a truth table for it. The result is below.

p	q	$p \vee q$	$\neg q$	p
T	T	T	F	T
T	F	T	T	T
F	T	F	F	F
F	F	F	T	F

The truth table can be used to show that this argument is valid. To see how, note that there is only one assignment of truth values to the sentence letters of this argument which makes both of the premises true: the second assignment, that is, the assignment in the third row. That assignment also makes the conclusion true. Therefore, for each assignment of truth values to the sentence letters of this argument which makes both of the premises true, that assignment also makes the conclusion true. So by definition, this argument is valid.

Therefore, by the definition of validity for arguments in English, the iron argument is also valid. In other words, the iron argument is valid because when translated into L , the resulting formal argument is valid. So the original, untranslated argument is valid too.

Now let us see whether the premises of the iron argument are true. Indeed, as a quick online search shows, they are. It is true that iron oxide is either red or

yellow; that is a simple fact about chemistry. It is also true that iron oxide is not yellow; that is another chemistry fact.

Therefore, the iron argument is sound. For it is valid, and all of its premises are true.

1.3 More Examples

In this section, I work through a few more examples of valid arguments and sound arguments. Consider the following argument in English.

1. Sara will go to the party if and only if Avon will.
2. Avon will go to the party.
3. Sara will go to the party.

Call this the ‘party argument’.

Let us see whether the party argument argument is valid. To do that, we must translate the party argument into an argument in L . The result is below.

1. $p \leftrightarrow q$
2. q
3. p

In this translation, ‘ p ’ represents the sentence “Sara will go to the party”, and ‘ q ’ represents the sentence “Avon will go to the party”.

To see that this argument is valid, consider the corresponding truth table below.

p	q	$p \leftrightarrow q$	q	p
T	T	T	T	T
T	F	F	F	T
F	T	F	T	F
F	F	T	F	F

As a simple check confirms, for each assignment of truth values to sentence letters which makes *all* of the premises in the argument true, that assignment *also* makes the conclusion true. In particular, there is just one assignment of truth values to sentence letters which makes all of the premises in the argument true: the first assignment; that is, the one in the second row. Each other assignment makes at least one premise in the argument false. And that first assignment also makes the conclusion true. So by the definition of validity for arguments in L , the above argument is valid.

Therefore, by the definition of validity for arguments in English, the party argument is also valid. In other words, the party argument is valid because when translated into L , the resulting formal argument is valid. So the original, untranslated argument is valid too.

Now let us see whether the argument below is sound.

1. Iron oxide is either red or yellow.
2. Iron oxide is red.
3. Iron oxide is yellow.

Call this the ‘yellow argument’.

To start, let us see whether the yellow argument is valid. To do that, we must translate the yellow argument into an argument in L . The result is below.

1. $p \vee q$.
2. p
3. q

In this translation, ‘ p ’ represents the sentence “Iron oxide is red”, and ‘ q ’ represents the sentence “Iron oxide is yellow”.

To determine whether or not this argument is valid, construct a truth table for it. The result is below.

p	q	$p \vee q$	p	q
T	T	T	T	T
T	F	T	T	F
F	T	F	T	T
F	F	F	F	F

The truth table can be used to show that this argument is *invalid*. As a quick check reveals, it is *not* the case that for every assignment of truth values to sentence letters which makes both of the premises true, that assignment also makes the conclusion true. For there is an assignment of truth values to sentence letters which (i) makes both of the premises true, but (ii) makes the conclusion false. It is the second assignment; that is, the assignment in the third row of the truth table. On that assignment, both premise ‘ $p \vee q$ ’ and premise ‘ p ’ are true, but the conclusion ‘ q ’ is false. So the formal argument is invalid.

Therefore, by the definition of invalidity for arguments in English, the yellow argument is also invalid. In other words, the yellow argument is invalid because when translated into L , the resulting formal argument is invalid. So the original, untranslated argument is invalid too.

And therefore, by the definition of unsoundness for arguments in English, the yellow argument is unsound. For if an argument is invalid, then by definition, it is unsound. And the yellow argument is invalid.

Note that the yellow argument is unsound *even though* all of its premises are true. For true premises are not enough to make an argument sound. In order to be sound, an argument must satisfy two conditions: it must be valid, and it must have only true premises. The yellow argument does not satisfy the first condition, and so it is unsound.

As a final example, let us see whether the argument below is sound.

1. Iron oxide is either red or yellow.
2. Iron oxide is not red.
3. Iron oxide is yellow.

Call this the ‘red argument’.

To start, let us see whether the red argument is valid. To do that, we must translate the red argument into an argument in L . The result is below.

1. $p \vee q$.
2. $\neg p$
3. q

In this translation, ‘ p ’ represents the sentence “Iron oxide is red”, and ‘ q ’ represents the sentence “Iron oxide is yellow”.

To determine whether or not this argument is valid, construct a truth table for it. The result is below.

p	q	$p \vee q$	$\neg p$	q
T	T	T	F	T
T	F	T	F	F
F	T	F	T	T
F	F	F	T	F

The truth table can be used to show that this argument is valid. To see how, note that there is only one assignment of truth values to the sentence letters of this argument which makes both of the premises true: the third assignment, that is, the assignment in the fourth row. That assignment also makes the conclusion true. Therefore, for each assignment of truth values to the sentence letters of this argument which makes both of the premises true, that assignment also makes the conclusion true. So by definition, this argument is valid.

Therefore, by the definition of validity for arguments in English, the red argument is also valid. In other words, the red argument is valid because when translated into L , the resulting formal argument is valid. So the original, untranslated argument is valid too.

It is not the case, however, that both premises in the red argument are true. It is true that iron oxide is either red or yellow; that is a simple fact about chemistry. But it is false that iron oxide is not red. As mentioned earlier, iron oxide is red.

And therefore, by the definition of unsoundness for arguments in English, the red argument is unsound. For if an argument has a false premise, then by definition, it is unsound. And the red argument has a false premise.

Note that the yellow argument is unsound *even though* it is valid. For validity is not enough to make an argument sound. In order to be sound, an argument must satisfy two conditions: it must be valid, and it must have only true premises. The yellow argument does not satisfy the second condition, and so it is unsound.

2 The Value of Validity and Soundness

Validity and soundness are extremely, extremely important. They represent the two main ways that an argument can be good. So if you want your arguments to be good ones—and you should want that—then your arguments must be both valid and sound. Unsound arguments, and invalid arguments, are bad.

One might wonder: why? What is it about valid arguments that makes them so good? What is it about sound arguments that makes them so good too? Why are invalid arguments bad? And why are sound arguments bad?

To see why valid arguments are good, it helps to recall the intuitive notion of validity. Intuitively, in valid arguments, the premises and the conclusion all ‘fit’ together. A little more precisely: intuitively, in valid arguments, the truth of the premises guarantees the truth of the conclusion. *If* the premises in a valid argument happen to be true, then the conclusion *must* be true.

And that, in short, is why valid arguments are so valuable. If you have a valid argument for some claim, then in order to establish that claim, you need only show that the premises in your argument are true. For if your argument is valid, then the truth of the premises *guarantees* the truth of the conclusion. So if your premises hold, then your claim must hold too. In other words, to establish your conclusion, you need only establish your premises: for in a valid argument, the conclusion follows from the premises automatically.

Similarly for soundness. Intuitively, in sound arguments, the premises (i) ‘fit’ the conclusion, and (ii) are true. A little more precisely: intuitively, in sound arguments, the premises both (i) guarantee the conclusion, and (ii) actually hold. So if there is a sound argument for a conclusion, then that conclusion *must* be true. For in a sound argument, the premises are true and the argument is valid. So the conclusion is guaranteed.

And that is why sound arguments are so valuable. If you have a sound argument for some claim, then the claim holds. It is worth emphasizing this: the claim *absolutely must* hold, if the argument is sound. No one can truthfully deny it. For if the argument is sound, then the premises are true. And if the argument is sound, then the argument is valid: and in a valid argument, if the premises are true then the conclusion must be true. Therefore, if you have a sound argument for some claim, then the truth of the claim is guaranteed. It does not matter what anyone says, or thinks, or wants. The claim is simply true.

This is why logic is, I think, one of the most important subjects to study. Propositional logic is a theory of how we ought to reason. The best way to reason, according to that theory, is to use valid and sound arguments. The world would be a much better place, if people reasoned in the ways that propositional logic suggests. Not because people would be robots; they wouldn't be. The world would be a better place because people would not be so easily misled.

It is extremely, extremely important to be a good reasoner. If someone is bad at reasoning, then it is very easy to lead them astray. Politicians take advantage of this all the time: by lying, bullshitting, and so on. More precisely, what many politicians do is

- (i) say things that seem like arguments, but are actually non-arguments,
- (ii) give arguments which seem good, but are actually invalid,
- (iii) give arguments which—though valid—have false premises that are hidden away, and so are unsound.

If you study propositional logic, you will become better at detecting when politicians try to take advantage of you in these ways. Of course, there are many other ways in which politicians take advantage of the voting public. But this is an extremely important one. And it is one that you can, pretty easily, do something about.